Chapter 16

The Invizicube

We have all been conditioned from birth to believe that we know what 3D is… it's the 3-Dimensional world around us. It's *obvious* what it is. It's the physical universe, reaching out into the unimaginably vast expanse of outer space, and it possesses length, width and height which we can, if we like, measure in metres. Straightforward.

Because 3D possesses three degrees of freedom, we tend to think of 3-Dimensionality very uniformly in volume terms, so that a cubic metre of 3-Dimensionality would be the same throughout the universe, whether it is a cubic metre of rock or a cubic metre of ocean, a cubic metre of the sun or a cubic metre of outer space. If we take the solar system, all we need do is draw an imaginary box-grid over it. It then becomes a small step for man to extend that grid, and simply pop the whole universe into it.

Now, I know that scientists are fully aware that this box-grid approach to the universe may not be accurate at larger scales, but it does represent the way we tend to view the world on a daily basis, *'For Nature,'* as Newton observed, *'is very consonant and conformable to herself'* [a] and we are inclined to accept her at face value, but the world as we experience it and the world as it actually is are two different things. The world as we perceive it is the world of Newton – unambiguous, fixed, and 3D – where everything happens against the backdrop of space and time. But the real world is the world of Einstein. The universe is in reality not 3-Dimensional, it is known to be 4-Dimensional.

Minkowski's Shadow

With characteristic humility[b], Einstein wrote concerning Special Relativity that,

> *'In the first place we must guard against the opinion that the four-dimensionality of reality has been newly introduced for the first time by this theory.'* [c]

For eight centuries the once-great Inca civilisation of America – although not possessing the sophistication of the modern age – passed down the relationship of space and time as a single concept; however (although I doubt Einstein was referring to them) they were the exception. Historically the earliest European mention of space-time as a mathematical concept was in 1754 by Jean le Rond d'Alembert in that imposing record of the Enlightenment – the 20,000,000 word *Encyclopédie*[d]. But it was Einstein's former teacher, Hermann Minkowski – the man who introduced him to a geometrically 4D understanding of the universe – who reached up into the world of philosophy to grab the hot air balloon of space-time and tether it to the ground of science, stating in 1908 that,

> *"The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."*

How right he was, and in return they named a universe after him.

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[a] Sir Isaac Newton, *Opticks, Query 31*, Cosimo 2007

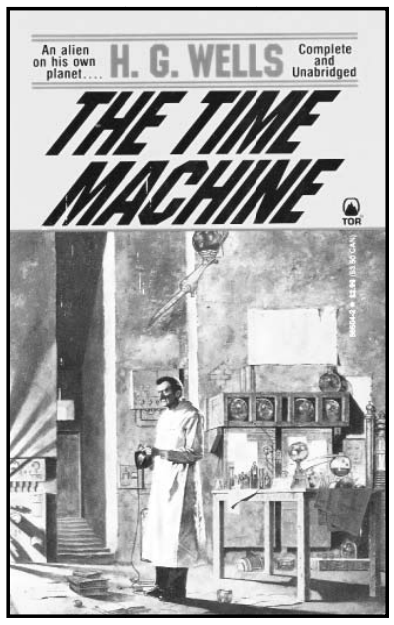
[b] And in stark contrast to many of today's aspiring groundbreakers for whom the first thing to make sure of is that their thing gets named after them!

[c] Albert Einstein, *Relativity, Appendix 5*, Routledge 2001, P151

[d] *'It sought not only to give information, but to guide opinion,'* so said the *Encyclopædia Britannica* in 1911.

When is a Cube Not a Cube?

But the implications of 4D for our worldview may run deeper than we think. Ten years before Einstein, visionary writer HG Wells exposed the logical consequences of isolating the 3rd Dimension in the first chapter of his classic novella, *The Time Machine*. In the story, the Time Traveller explains to his friends...



"You know of course that a mathematical line, a line of thickness ***nil****, has no real existence. They taught you that? Neither has a mathematical plane. These things are mere abstractions."*

"That is all right," said the Psychologist.

"Nor, having only length, breadth, and thickness, can a cube have a real existence."

"There I object," said Filby. "Of course a solid body may exist. All real things--"

"So most people think. But wait a moment. Can an ***instantaneous*** *cube exist?"*

"Don't follow you," said Filby.

"Can a cube that does not last for any time at all, have a real existence?"

Filby became pensive.

"Clearly," the Time Traveller proceeded, "any real body must have extension in ***four*** *directions: it must have Length, Breadth, Thickness, and--Duration. There are really four dimensions, three which we call the three planes of Space, and a fourth, Time. There is, however, a tendency to draw an unreal distinction between the former three dimensions and the latter..."*

Writing a decade before Special Relativity, Wells had the foresight to express a profound truth: a cube is actually 4-Dimensional. If one measures out a cubic metre of our universe it is not a cubic metre of space, but a cubic metre in *space-time*.

How therefore, if it is impossible to *isolate* the 3rd Dimension from time, is it possible for us to know whether 'space' is actually 3-Dimensional in any true and accurate sense? How can we even be sure that anything we are looking at is actually a 3-Dimensional object? In Minkowski's words, how can it *'preserve an independent reality'*? Could it be that our handy everyday concept of 3D is nothing more than a *representation* of the 3rd Dimension within the 4th *in exactly the same way as the plane, the line and the point?*

Since, as Wells reminded us, we cannot stop time, no-one has ever seen 3D except within a 4D setting. Therefore, as an isolated entity with an independent reality, ***no-one has ever seen 3D***.

Edge-On

Just because we can't actually see the 4th Dimension does not mean it doesn't influence the way the universe appears[a]. Indeed it *dictates* how the universe appears. It is the 4-Dimensional 'time-shape' of our universe that renders it 3-Dimensional to our experience. How can this be?

[a] I.e. manifests to the senses.

It's all a question of viewpoint. We *perceive* the universe as 3-Dimensional because we view it 'edge-on'. As we saw earlier, just as the surface of a 3D ball is 2D, the universe's apparent 3-Dimensionality derives from the fact that every time we open our eyes we are looking out at the boundary or surface of a 4D hypersphere, so although the thing we are *actually* looking at is 4D, it only *appears* 3D because of our 'edge-on' (one dimension lower) viewpoint. In Chapter 3 this was named as the *'Edge-On' Principle*[a], as it was derived from the simple geometry of *Flatland* wherein *A Square* views his 2-Dimensional world edge-on in 1D.

Put another way, we are not seeing and experiencing a 3-Dimensional universe; we are seeing and experiencing a 4-Dimensional universe, one dimension lower. It's just so easy to forget that we inhabit space-time. The truth is… we can't even *begin* to know what a '3-Dimensional universe' might look like, because, to our knowledge, none exists. But we have all assumed that we do. This may sound like an exercise in geometrical pedantry, but it is not and I will explain why…

Drawing the Line

It is generally accepted scientifically (and common sensically) that whilst the three spatial dimensions of our world – the 'usual three' of length, width and height – cannot exist independently, they do co-exist in 3D form. This is a fundamental axiom of science. However, let's look more closely at this all-too-obvious assumption (yes, assumption)…

As we have already seen, the *line* and the *plane* do not actually exist out there in the universe, only in our heads. They are simply *representations* of 1- and 2-Dimensionality proper, whose existence, inexplicably, we can very easily imagine. Dimensionally our cubic metre of rock is composed of stacked-up planes (*representations* of 2D) which in turn are composed of stacked-up lines (r*epresentations* of 1D). Now, because lines and planes do not actually exist except as mathematical concepts, this leads us directly to the question: ***How can something (the cube) that is made from things that don't exist, exist?***

What I am suggesting here is that it doesn't.

Or as HG put it, *"a mathematical line, a line of thickness* nil, *has no real existence… Neither has a mathematical plane. These things are mere abstractions… Nor, having only length, breadth, and thickness, can a cube have a real existence."* This is a perfectly logical conclusion, and in a sense disturbingly obvious. Without *'Duration'*, there is no difference whatsoever between a line, a plane, and all the physicality of the world.

Of course it's not that the cube of rock doesn't *physically* exist. It clearly does, (although at the quantum level the very term 'physical' dissolves into a bewildering array of particles, waves, forces, charges, probability and open space) but the point I am making is that its 'length, width and height 3-Dimensionality' may simply be a *representation* of a 3rd Dimension, proper.

In other words, length, width and height do not constitute an actual description or measurement of our 1m³ of rock's 3-Dimensionality, merely a representation. This would mean that the spatial dimensions as understood by science are not 'the spatial dimensions' at all, but merely representations, placeholders, shadows of something else, whose inherent properties and basic inter-relations they embody.

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[a] *The 'Edge-On' Principle:* Each dimension is viewed from within itself one dimension lower.

The Search for Invariance

To summarise… the line and the plane are *representations* of true 1- and 2-Dimensionality whose existence we can very easily imagine. In precisely the same way, we are *imagining* the existence of a 3-Dimensionality as represented by the 3D space produced in our experience by the addition of *height*, because, similar to the line and the plane, what we think of as the cube of rock's 3-Dimensionality (length, width and height) is nothing more than a *representation* of the 3rd Dimension within our experience of the 4D universe. We have all been fooled by its existence!

But how does this happen?

The reason we accept the world around us as 3D is because our bodies – being part of the world – are also 3D, and our senses operate as mediators between our consciousness and the world. However, in tandem with the strong force, it is the electromagnetic force that causes 'solidity', not 3-Dimensionality. Our hands experience an object as solid because the periphery of both hand and object share the same negative charge, and 'like' charges repel. This is why we perceive 3D volume as 'real' whilst 2D planes etc seem mere abstractions. If our hands were 2D we would recognise 2D – and be in Spaceland! Over the past few chapters we saw how all the 'stuff' in the universe hardly amounts to a hazelnut. We see and feel a world that is largely not there, because all we see and feel are *forces* and *energy* upon which we superimpose geometrical concepts, *only one of which is 3D!*

Reflection… Our bodies share the same space, are made of the same stuff, and exist as part and parcel of the 3-sphere around us, which is the visible boundary of the 4D hypersphere that is our amazing 'time-shaped' universe. But being made of the same stuff does not make 3D *automatically* more real than 2D. All it tells us is where our dimensional viewpoint lies.

This is not at all easy to get one's head around, but it is where the geometry leads. If we find this logic difficult to accept we are going to have to consider… do we follow HG Wells' character *Filby* and declare *"There I object,"* going with 3-Dimensionality the way we find it? Or do we go with the maths, which exhibits no specific bias toward one spatial dimension over another?

Brian Cox and Jeff Forshaw again,

> *'Perhaps we will never understand the true nature of the relationship between mathematics and nature, but history has shown that mathematics allows us to organise our thinking in a way that proves to be a reliable guide to a deeper understanding."*[a]
>
> *'Identifying the invariant properties is far from easy, however, because nature's underlying simplicity and beauty are often hidden.'* [b]

Without the discipline to approach the world in this way, our understanding of nature would still be Aristotelian. For example, without the figures who would ever have accepted it's the Earth that goes round the Sun? Or worse... like the learned Sir Bedevere on the eve of his quest for the Holy Grail[c], those wise in the ways of science might still be employing sheep's bladders to prevent earthquakes!

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[a] Brian Cox and Jeff Forshaw, *Why Does $E=mc^2$?*, Da Capo Press 2010, P63
[b] Ibid., P61
[c] *Monty Python and the Holy Grail*, 1975

Age of Reason

We use the concept of time, or the block universe, to describe the 4th Dimension. In the same way, does there perhaps exist a more concrete way to describe the 3rd Dimension, proper? But first – before we pick up the dimensional trail again – let's put in place some logic to clarify what 3D is *not*…

We have seen that, by the *Principle of Stacking*[a],

- We mentally construct a line from points which do not exist in the real world, and the line doesn't exist either.
- We then mentally construct a plane from lines which do not exist, and the plane doesn't exist either.
- We then mentally construct 3D space from planes which do not exist.

It is not the science fiction of Wells, but *reason* that demonstrates that ***3-Dimensional space takes precisely the same form as the point, the line and the plane***.

Gottfried Wilhelm von Leibnitz, 1646-1716

The Problem of Space

You may still believe this to be a moot point, but the essential nature and existence or otherwise of space is a deep-rooted problem which has been debated by philosophers, and has left scientists with a sense of unease, from time immemorial. One such was Newton's arch-nemesis, the German mathematician, philosopher, all-round genius and co-wearer of giant wigs, Gottfried Leibniz, who maintained that,

'Space has no reality apart from material things; it is nothing more than an abstract, mathematical description of relations that hold between objects.' [b]

Albert Einstein was a great scientist, but he was also a highly respected philosopher of science, and following on from Plato, Descartes, Leibniz, Newton and Kant, he gave the nature of space a lot of thought…

> *'The psychological origin of the idea of space, or of the necessity for it, is far from being so obvious as it may appear to be on the basis of our customary habit of thought. The old geometers deal with conceptual objects (straight line, point, surface), but not really with space as such,'*

Einstein wrote this in 1952, just three years before he died, in a final addendum to his original 1916 discourse on Relativity which he titled *'Relativity and the Problem of Space'*.[c] Here is what he said,

> *'Newton himself and his most critical contemporaries felt it to be disturbing that one had to ascribe physical reality both to space itself as well as to its state of motion; but there was at that time no other alternative, if one wished to ascribe to mechanics a clear meaning.'*

In other words, everything (for Newton) had to be moving relative to some fixed, grid-like backdrop.

Continuing…

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[a] *The Principle of Stacking:* Each dimension is composed of an indefinitely high number of cross-sections (slices) of the dimension below, stacked together and fused into a single entity.

[b] Martin Gardner, *The Ambidextrous Universe*, Pelican 1970, P159

[c] Albert Einstein, *Relativity, Appendix 5*, Routledge 2001

> *'It is indeed an exacting requirement to have to ascribe physical reality to space in general, and especially to empty space. Time and again since remotest times philosophers have resisted such a presumption. Descartes argued somewhat on these lines: space is identical with extension, but extension is connected with bodies; thus there is no space without bodies and hence no empty space.'*

Continuing…

> *'… the general theory of relativity confirms Descartes' conception in a roundabout way. What brought Descartes to his remarkably attractive view was certainly the feeling that, without compelling necessity, one ought not to ascribe reality to a thing like space, which is not capable of being "directly experienced".'*

In a nutshell, what Einstein is saying is that, following on from Descartes (and Leibniz), the apparent 3-Dimensionality of empty space has no objective, independent existence, because space can only be defined in terms of the things (people, planets, etc) that it is the empty bit between, which he tells us *'in a roundabout way'* General Relativity confirms. Describing this view as *'remarkably attractive'*, Einstein marvels that the old geometers failed to realise that space belongs with the other '*conceptual objects (straight line, point, surface)'*. Space, according to Einstein, is no different to the line, the point and the plane.

This is no moot point, and the answer does not lie in a physics that is based on the 'usual three'.

What's in a Word?

So, to put 3D into its true perspective it might help to state the situation using parallel statements:

- A point is a representation of the 0^{th} Dimension as it appears within the 4D universe.
- A line is a representation of the 1^{st} Dimension as it appears within the 4D universe.
- A plane is a representation of the 2^{nd} Dimension as it appears within the 4D universe.
- An invizicube is a representation of the 3^{rd} Dimension as it appears within the 4D universe.

Oops, sorry! As far as I know there doesn't exist a word in the English language to describe a 3D entity having length, width and height *that is an abstract representation of the 3^{rd} Dimension within the 4D universe*, so I've invented one… *the invizicube.*

So now we have… the point, the line, the plane and the invizicube.

Eureka! 2.0

By this reasoning we see how volume *per se* cannot be a true measure of 3-Dimensionality. Instead volume itself is a representation – a mentally useful, working expression of all the general principles that apply within the 3^{rd} Dimension proper, as they play out within a 4-Dimensional universe. Einstein concludes by saying that,

> *'On the basis of the general theory of relativity,.. space...has no separate existence.'* [a]

What we perceive as the length, width and height of the world around us is just a sensually integrated abstract representation of the 3^{rd} Dimension, proper – whatever that may be – with the invizicube as our

[a] Albert Einstein, *Relativity, Appendix 5*, Routledge 2001

handy abstract concept of that space; a space whose only role is to describe the relationships between ‘things’. Einstein also points out that,

> *‘space and time must be regarded as a four-dimensional continuum that is objectively unresolvable’.*

In pursuit of physical reality it is the four-dimensional continuum with which we have to do, whilst – in *Flatland* dimensional terms – the ‘unresolvability’[a] of the four dimensions within the continuum is the manifestation around and within us of a nested hierarchy comprising the indissoluble stacking-up[b] of one dimension to become the next – a concept with which the old geometers would, I’m sure, have had no problem.

[a] *The Principle of Character:* Once the stacking of a dimension is complete it assumes a whole new character. Its individual cross-sections fuse together and their discrete nature becomes indiscernible.

[b] *The Principle of Stacking:* Each dimension is composed of an indefinitely high number of cross-sections (slices) of the dimension below, stacked together and fused into a single entity.